Scaling Analysis in Modeling Transport and Reaction Processes

By William B. Krantz. Wiley-InterScience, 2007.

Scaling analysis is an extremely valuable tool in modeling transport processes, where creating tractable sets of differential equations can be a major challenge. Put simply, the objective is to deduce the orders of magnitude of key quantities by careful examination of the governing equations, so that useful simplifications can be identified before attempting analytical or numerical solutions. Although widespread in the research literature, scaling has received little attention in textbooks, even at the graduate level. By providing the first book-length treatment of the subject, the author has done much to rectify that problem. The approach to the subject is very systematic, and the methodology is illustrated by a variety of examples involving fluid mechanics, heat transfer, mass transfer, and chemical reactions.

Following brief introductory comments in Chapter 1, a general procedure for scaling analysis is proposed in Chapter 2, consisting of a series of eight steps. Chapters 3–5 illustrate the methodology using mostly conventional examples drawn from fluid mechanics, heat transfer, and mass transfer, respectively. Chapter 6 focuses on reactive systems, going beyond the usual boundaries of transport texts to discuss aspects of reactor design. Chapter 7 provides several detailed case studies that illustrate applications of scaling analysis to process design. Whereas Chapter 2 is needed to appreciate the subsequent material, the applications chapters may be read in any order if one already has a good grounding in transport analysis in chemical engineering.

The unique focus on scaling, the well-organized approach, and the number and variety of worked examples, will do much to remove the mystery that some associate with approximation methods. However, this is not a textbook for undergraduates and would require considerable supplementation even for beginning graduate students. The reader must be familiar already with the conservation equations for mass, momentum, energy, and species, which are presented in appendices without derivations. A "vocabulary" of standard one-dimensional (1-D) examples (ones involving ordinary differential equations) is presumed. A firm understanding of how to formulate boundary value problems involving partial differential equations is also needed. Finally, without some experience in solving such problems, the benefits of the simplification process might not be apparent. That is, to motivate such efforts, the reader must have an understanding of what constitutes an "easy" or "difficult" problem.

The reasoning used tends to be presented clearly, although there are occasional pitfalls with the approach. For example, in the discussion of the laminar boundary layer approximation in section 3.4, the example chosen is a flat plate oriented parallel to the approaching stream. Whereas the formal procedure that is used suggests that a large Reynolds number (Re) is sufficient to make the pressure gradient in the flow direction negligible, in actuality it is the combination of large Re and a perfectly streamlined object (i.e., a thin, flat plate) that does this, as is well known. Applying the same formal reasoning to flow past a more blunt object, where the pressure gradient would not be negligible, would lead to an erroneous conclusion. What is missing is a consideration of the interaction between the outer (inviscid) flow and the shape of the submerged object. An error crops up also in the discussion of flow through a converging, flat-wall channel in section 3.3. It is concluded there that Re times the width-to-length ratio must be small to give creeping flow, which would seem to preclude the possibility of creeping flow through a very short channel (i.e., an orifice). Actually, $\mathrm{Re} \ll 1$ is sufficient. The problem in this case is that the choice of scales in the analysis implicitly assumed that the channel was long.

Two cautionary conclusions may be drawn from these examples, and from others where the approximations and error estimates are entirely appropriate. One is that the scaling procedure must be applied with great care. The other is that a certain amount of physical intuition is needed to ensure success. In this context "intuition" is simply experience with similar problems that has been distilled into a set of expectations. Such experience will lead to a hypothesis concerning the dominant rate processes, one which can be tested using the formal scaling procedure. If that test reveals no contradictions, the hypothesis is likely to be correct; if contradictions are evident, a new hypothesis is needed. Obviously, if experience leads to an initial view of the problem that is already qualitatively correct, the rest of the analysis will go much more smoothly.

It would have been good if problems that involve Neumann (e.g., specified flux) boundary conditions had received more attention. They tend to present a richer set of challenges than those with only Dirichlet (e.g., specified temperature) boundary conditions, in that fewer scales can be assigned at the outset. However, the range of worked examples is impressive as is.

In summary, this book is recommended as a resource for anyone seeking a systematic approach to the simplification of transport models. Its focus is unique, and it squarely addresses an educational need. For an advanced graduate-level transport course, especially, it would be a good source of worked examples and practice problems.

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AiChE Journal, Vol. 54, 3028 (2008)

DOI 10.1002/aic.11681
Published online October 6, 2008 in Wiley InterScience (www.interscience.wiley.com).